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TECHNICAL NOTES

Improved calculation of the steady-state heat conduction from/towards a cylinder in the centre of a slab†

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IN THE literature the Nusselt number, *Nu*,‡ for steady-state conduction of heat from/towards a cylinder has been given for several enclosures of a cylinder with diameter D [1–3]. In Fig. 1 these *Nu* are given for a few similar geometries. Here, the dimension not drawn should be considered as being infinite. As indicated, the Nusselt number for a cylinder in a slit would be larger than that with a quadrangular enclosure. This is implausible. Because the theoretical derivation of *Nu* could not be traced in the slit case, an estimate for the Nusselt number in the cylinder–slit case will be derived here.

The result given by Carslaw and Jaeger [4] is taken for the stationary temperature distribution ($T(x,y,z)$), as caused by a continuously acting point source in (x_0, y_0, z_0) with strength $q/(\rho C_p)$, where (x,y,z) and (x_0,y_0,z_0) are points in the space between the planes $z = 0$ and L, these planes being kept at temperature $T_2 = 0$ as the reference temperature

$$T(x,y,z) = \frac{q}{\pi\lambda L}\sum_{m=1}^{\infty} \sin\left\{\frac{m\pi z}{L}\right\}\sin\left\{\frac{m\pi z_0}{L}\right\}K_0\left(\frac{m\pi R}{L}\right) \quad (1)$$

where K_0 is the zeroth-order modified Bessel function of the second kind, where $R = \{(x-x_0)^2+(y-y_0)^2\}^{1/2}$, ρ the density, C_p the specific heat and q the heat generated by the source per time.

The temperature at $(x, 0, z)$ due to a continuous line source at $(0, -\infty < y_0 < \infty, L/2)$ with strength $q/(\rho C_p)$ follows from integration of equation (1) over y_0

$$T(x,0,z) = \frac{2q}{\pi\lambda L}\sum_{n=0}^{\infty} \sin\left\{\frac{(2n+1)\pi z}{L}\right\}\sin\left\{\frac{(2n+1)\pi}{2}\right\} \times \int_0^{\infty} K_0\left(\frac{(2n+1)\pi(x^2+y_0^2)^{1/2}}{L}\right)\mathrm{d}y_0. \quad (2)$$

Here, q is the amount of heat generated per time per length. With the aid of this expression the temperature has been calculated at three positions at a distance $D/2$ from the line source. These positions are $(0, 0, L/2+D/2)$, $(D/\sqrt{8}, 0, L/2+D/\sqrt{8})$ and $(D/2, 0, L/2)$. In the calculations use was made of the integral representation of K_0

$$K_0(x) = \tfrac{1}{2}\int_0^{\infty} (1/p)\exp\left\{-p-\frac{x^2}{4p}\right\}\mathrm{d}p. \quad (3)$$

The results are

$$T(0,0,L/2+D/2) = \frac{q}{\pi\lambda}\sum_{n=0}^{\infty}\frac{1}{(2n+1)}\cos\left\{\frac{(2n+1)\pi D}{2L}\right\} \quad (4)$$

$$T(D/\sqrt{8},0,L/2+D/\sqrt{8}) = \frac{q}{\pi\lambda}\sum_{n=0}^{\infty}\frac{1}{(2n+1)} \times \exp\left\{-\frac{(2n+1)\pi D}{L\sqrt{8}}\right\}\cos\left\{\frac{(2n+1)\pi D}{L\sqrt{8}}\right\} \quad (5)$$

and

$$T(D/2,0,L/2) = \frac{q}{\pi\lambda}\sum_{n=0}^{\infty}\frac{1}{(2n+1)}\exp\left\{-\frac{(2n+1)\pi D}{2L}\right\}. \quad (6)$$

The behaviour of these three expressions is given graphically in Fig. 2. It is clear from this figure that the temperature gradient around a line source in the middle of a slit, at small values of D/L is only dependent on the distance to the source (D/L) and not on the radial direction with respect to the line source. The same holds if one replaces the line source by a cylindrical source with diameter D_i in the centre of the slit,

† This work is part of: J. Laven, Non-isothermal capillary flow of plastics related to their thermal and rheological properties, Doctoral Thesis, Delft University Press (1985).

‡ The Nusselt number, *Nu*, which essentially is a dimensionless temperature gradient, is defined here as $Nu = q/\{\pi\lambda(T_1 - T_2)\}$ where T_1 and T_2 are the temperatures of the inner and outer walls of the two-dimensional geometry. The quantity q represents the amount of heat, per time and per length from the infinite central cylinder. The heat conductivity of the medium in the interspace is denoted by λ.

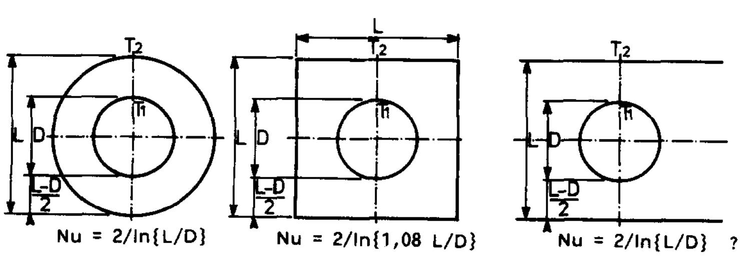

FIG. 1. Several possibilities for symmetrical enclosures of cylinders and corresponding Nusselt numbers according to the literature. The dimension not drawn should be considered to be infinite. The temperatures of the cylinders and of the enclosures are uniform and constant.

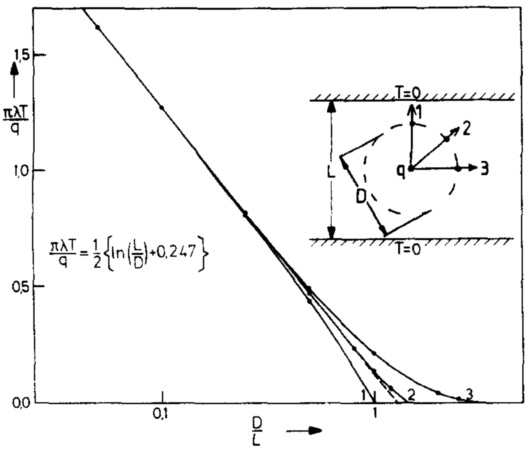

FIG. 2. Steady-state temperature for three positions at a distance $D/2$ from a continuous line source with strength $1/(\rho C_p)$, the source being positioned in the middle between two plates at zero relative temperature.

provided D_i/L is sufficiently small, i.e. <0.1. Consequently, the heat transfer is direction independent in that case as well. From the slope of the straight line in Fig. 2 it is concluded that

$$Nu = hD_i/\lambda = q/\{\pi\lambda T_1\} = 2/\ln\{1.28L/D\}.$$

This result fits much better in the expected order of decreasing Nusselt numbers for a cylinder, a quadrangular tube and a slit as enclosures.

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Effect of boundary conditions at the lateral walls on the thermal entry lengths of horizontal CVD reactors

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1. INTRODUCTION

THE PREPARATION of semi-conductors, insulators and metals from vapor deposition has lately gained increasing importance. In the production of these films for electronic and optical devices, open flow systems have become of particular interest [1]. The interest in flow profiles and possible flow instabilities stems from the need to find conditions which will improve the uniformity of the deposit. Many configurations for CVD reactors have been proposed and are in use nowadays. For instance, the horizontal CVD reactor with its simple configuration constitutes one of the reactors most amenable for the analysis of flow phenomena. Although these reactors are currently used primarily for research and special applications, they still play an important role [2]. Upon entering the heated susceptor region of the reactor, the fluid starts to heat up and will develop a new linear profile (in the absence of natural convection) within its thermal entry length, x_D. It can be shown that thermal instabilities (that is, secondary flows) will be present in this development region for all non-zero Rayleigh numbers. Indeed, two-dimensional numerical results for the GaAs hot-wall reactor showed that rather small temperature differences between successive isothermal zones can cause back-flow against the imposed forced flow [3]. These secondary flows will be confined to the entrance or will exist throughout the reactor [4]. Numerical studies by Cheng *et al.* [5], Ou *et al.* [6] and Cheng and Ou [7] showed that for the case of large Prandtl number (Pr) fluids, the inclusion of secondary flow always lead to thermal entry lengths which are shorter than lengths which are determined in the Graetz fashion (neglecting secondary flows). In this work we assume that these numerical results will hold qualitatively for gaseous systems.

The importance of knowing this distance, lies among other reasons, on the temperature distribution that will be responsible for the distribution of homogeneous gas-phase reactions, besides governing transport and physical fluid properties. From a mathematical point of view, it is advantageous to know where the linear temperature profile is established since it is the basic state that will be perturbed to find critical values of the Rayleigh number in the developed region.

Several authors treated the limiting case of two infinite horizontal plates. Hsu [8] solved the case of a step increase in heat flux in both the top and bottom plates at $x = 0$. He considered axial conduction and showed that it is important for low Peclet numbers ($Pe < 45$). Hatton and Turton [9] studied the thermal development when the top plate is at ambient temperature and the bottom plate is heated. Cheng and Wu [10] studied the influence of axial conduction on thermal instabilities in the entrance region. Hwang and Cheng [11] considered a similar problem and found that for $Pr \geqslant 0.7$ the flow is thermally more stable in the thermal

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